

R_K

- ① If $ax^2+bx+c=0$ has exactly one Root $= 0$ then $c=0, b \neq 0$
- (2) If $ax^2+bx+c=0$ has both roots $= 0$ then $\boxed{b=0=c}, a \neq 0$
- (3) If $ax^2+bx+c=0$ has exactly 1 Root ∞ then $a=0, b \neq 0$
- (4) If $ax^2+bx+c=0$ has both Root ∞ then $a=b=0, c \neq 0$

Q $x = 3 + \sqrt{5}$ then

$$x^4 + 12x^3 + 44x^2 - 45x + 17 = 9$$

$$\underline{(x^2 - 6x + 4)(x^2 + 18x + 148) + 771x - 575}$$

$$x = 3 + \sqrt{5} \Rightarrow x - 3 = \sqrt{5} \Rightarrow 1551 + 771\sqrt{5}$$

$$(x-3)^2 = 5 \Rightarrow \underline{x^2 - 6x + 9 - 5 = 0}$$

$$\underline{x^2 - 6x + 4 = 0}$$

$$x^2 - 6x + 4 \overline{) x^4 + 12x^3 + 44x^2 - 45x + 17} \quad x^2 + 18x + 148$$

$$\underline{-x^4 + 6x^3 - 4x^2}$$

$$18x^3 + 40x^2 - 45x$$

$$\underline{-18x^3 + 108x^2 - 72x}$$

$$148x^2 - 117x + 17$$

$$\underline{148x^2 + 888x + 592}$$

$$771x - 575$$

Q₂ Find all Integral values of $a \rightarrow a \in \mathbb{Z}$
 for which $\text{QEqn} (x-a)(x-10)+1=0$
 has Integral Roots $x \in \mathbb{Z}$.

$$(x-a)(x-10) = -1$$

$$(I-a)(I-10) = -1$$

$$I \times I = -1$$

$x-a=1 \quad x-10=-1$ $x=9$ $9-a=1$ $a=8$	$x-a=-1 \quad x-10=1$ $x=11$ $11-a=-1$ $a=12$
--	--

$a=12, 8$

Q₃ If roots of $x^2+2ax+b=0$ are Real.
 $\alpha+\beta=-2a$
 $\alpha \cdot \beta = b$
 & distinct & they differ by at
 most $2k$ then $b \in ?$
 $\rightarrow D > 0 \quad \& \quad |\alpha - \beta| \leq 2k$
 $4a^2 - 4b > 0 \quad | \quad (\alpha - \beta)^2 \leq 4k^2$
 $a^2 > b \quad | \quad (\alpha + \beta)^2 - 4\alpha\beta \leq 4k^2$
 $\quad \quad \quad | \quad 4a^2 - 4b \leq 4k^2$
 $\quad \quad \quad | \quad a^2 - k^2 \leq b$
 $a^2 - k^2 \leq b < a^2$
 $b \in [a^2 - k^2, a^2)$

Q4 If one root of $x^2 - x - k = 0$ is sq^r of other than $k = ?$

$$x^2 - x - k = 0 \rightarrow x^2$$

$$\alpha + \alpha^2 = 1 \quad \left| \quad \alpha \cdot \alpha^2 = -k \right.$$

$$\alpha^3 = -k$$

M2

$$a^2(c + ac^2 + b^3) = 3abc$$

$$1^2 \cdot (-k) + (1) \cdot (-k)^2 + (-1)^3 = 3 \times 1 \times -1 \times -k$$

$$k^2 - k - 1 = 3k$$

$$k^2 - 4k - 1 = 0$$

$$k = \frac{4 \pm \sqrt{16 + 4}}{2} = \frac{2 \pm \sqrt{5}}{2}$$

Q5 $f(x) = ax^2 + bx + c, a \neq 0$ $a, b, c \in \mathbb{I}$ Smpb.

$$\left[\begin{array}{l} f(1) = 0 \text{ \& } 50 < f(7) < 60; \text{ } 70 < f(8) < 80 \\ \text{find } f(10) = ? \end{array} \right.$$

$$a + b + c = 0$$

$$50 < f(7) < 60$$

$$50 < 49a + 7b + c < 60$$

$$50 < 48a + 6b < 60$$

$$8 \dots < 8a + b < 10$$

$$8a + b = 9$$

$$f(x) = 2x^2 - 7x + 5$$

$$f(10) = 200 - 70 + 5$$

$$70 < f(8) < 80$$

$$70 < 64a + 8b + c < 80$$

$$70 < 63a + 7b < 80$$

$$10 < 9a + b < 11 \dots$$

$$9a + b = 11$$

$$8a + b = 9$$

$$a = 2$$

$$b = -7$$

$$c = 5$$

Q6 P.T. Bi Quadratic Eqn

$(ax^2+bx+c)(ax^2+dx-c)$ must

have at least two real roots

$a, b, c, d \in \mathbb{R}$?

$$D_1 = b^2 - 4ac$$

$$D_2 = d^2 + 4ac$$

$$\underline{D_1 + D_2 = b^2 + d^2 \geq 0}$$

Socho
Kya Karen?
→ Add

Dunia K Koi b 2 No add ho K +ve de Rhe hain!!

It means $2 + 2\pi$ $1 + \text{ve}$ $\text{all } \sqrt{\text{all } \text{all } \text{all}}$

lets $D_1 > 0 \Rightarrow 2$ Real Roots Ayenge hi.

Q If $a, b, c \in \mathbb{Q}$ P.T. Roots

$(b+(-2a))x^2 + (c+a-2b)x + (a+b-2c) = 0$
are also Rational?

Cyclic Order $\rightarrow \alpha = 1 = \text{Rational}$

$\beta = \text{Irr}$ Nahi ho Sakta

Irr. Roots Comes in BOHO

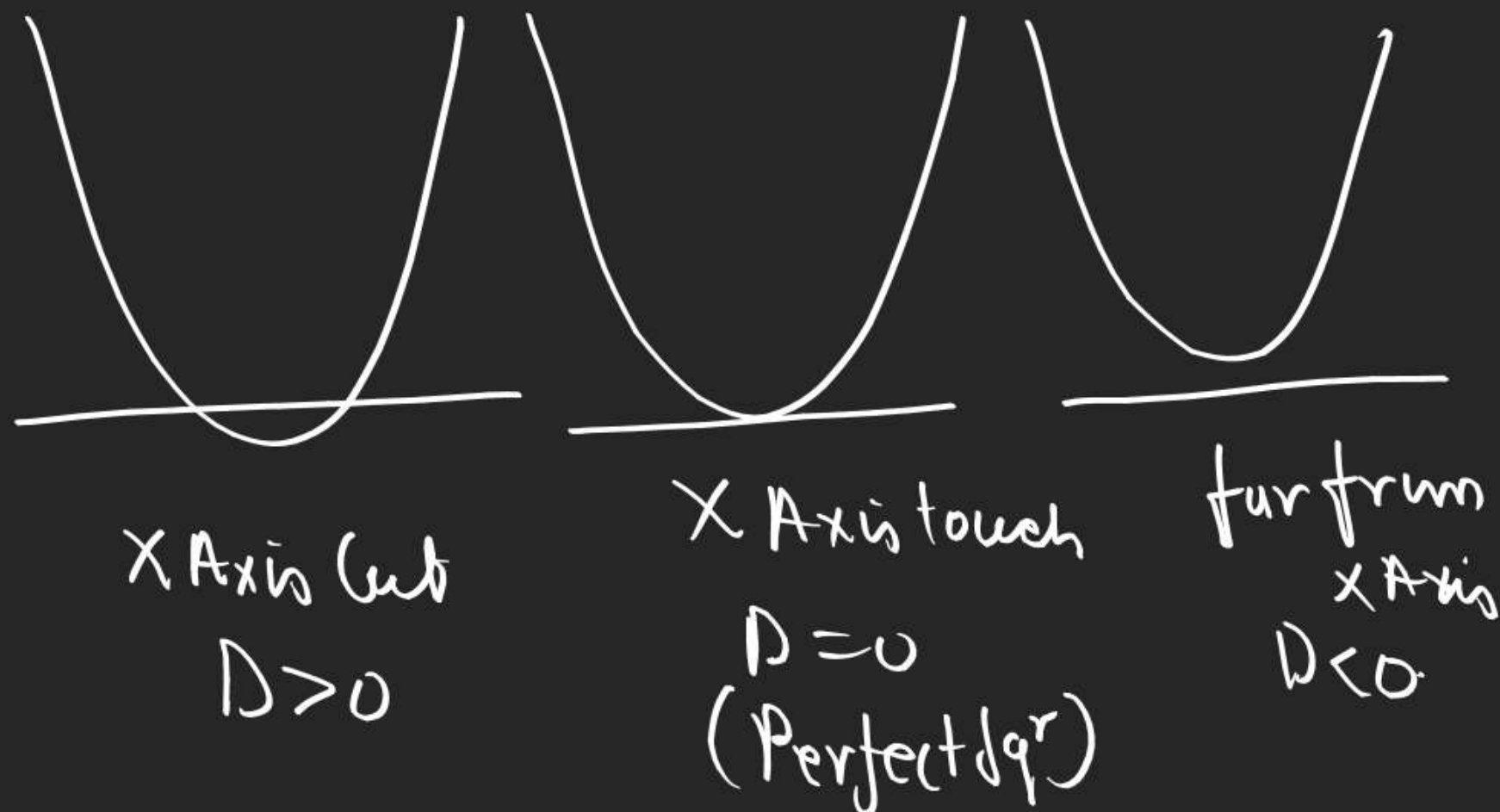
offa. (Pairs)

$\therefore \beta$ has to be Rational

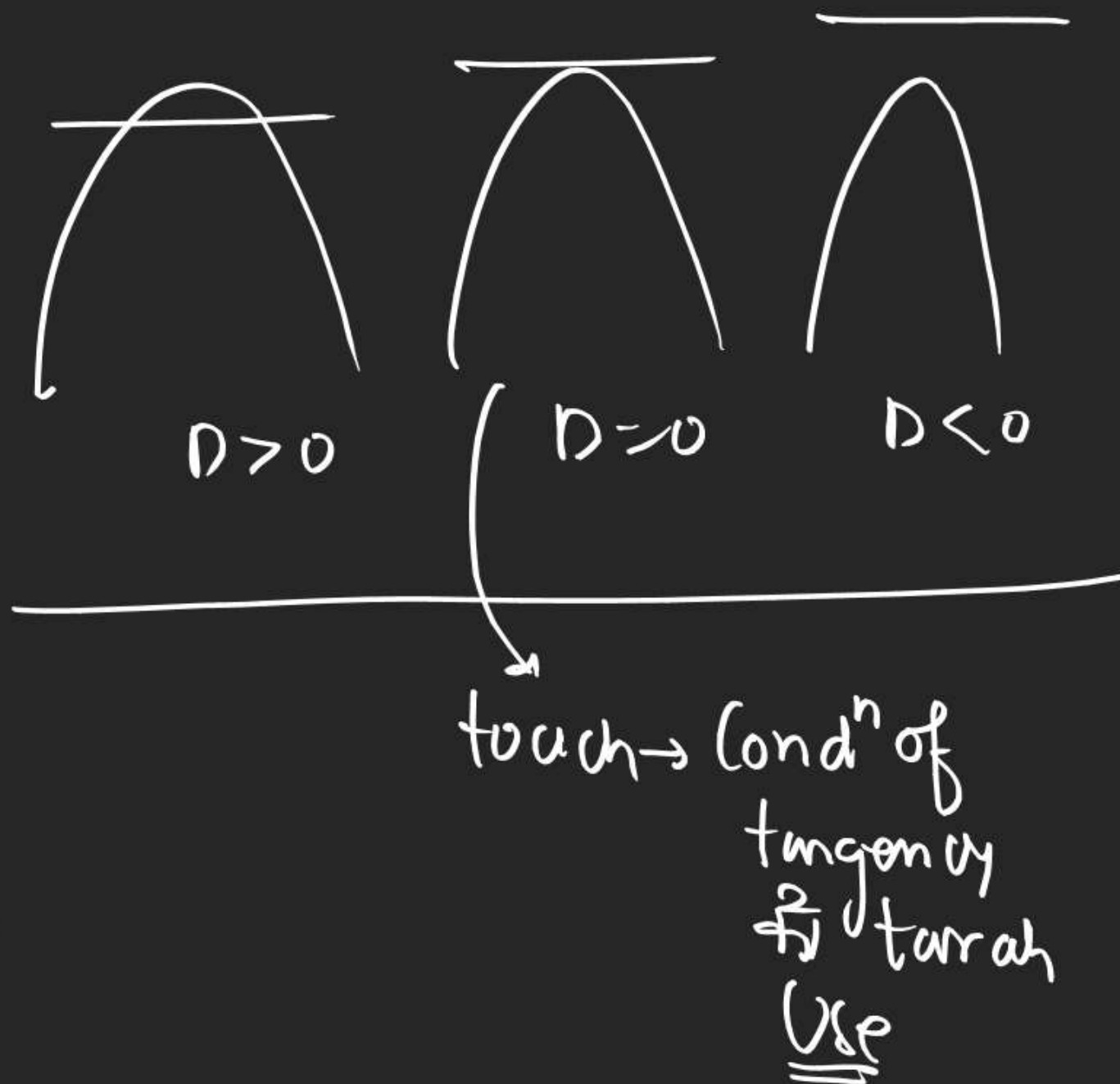
Both Roots Rational

Condition of Graph WRT "D"

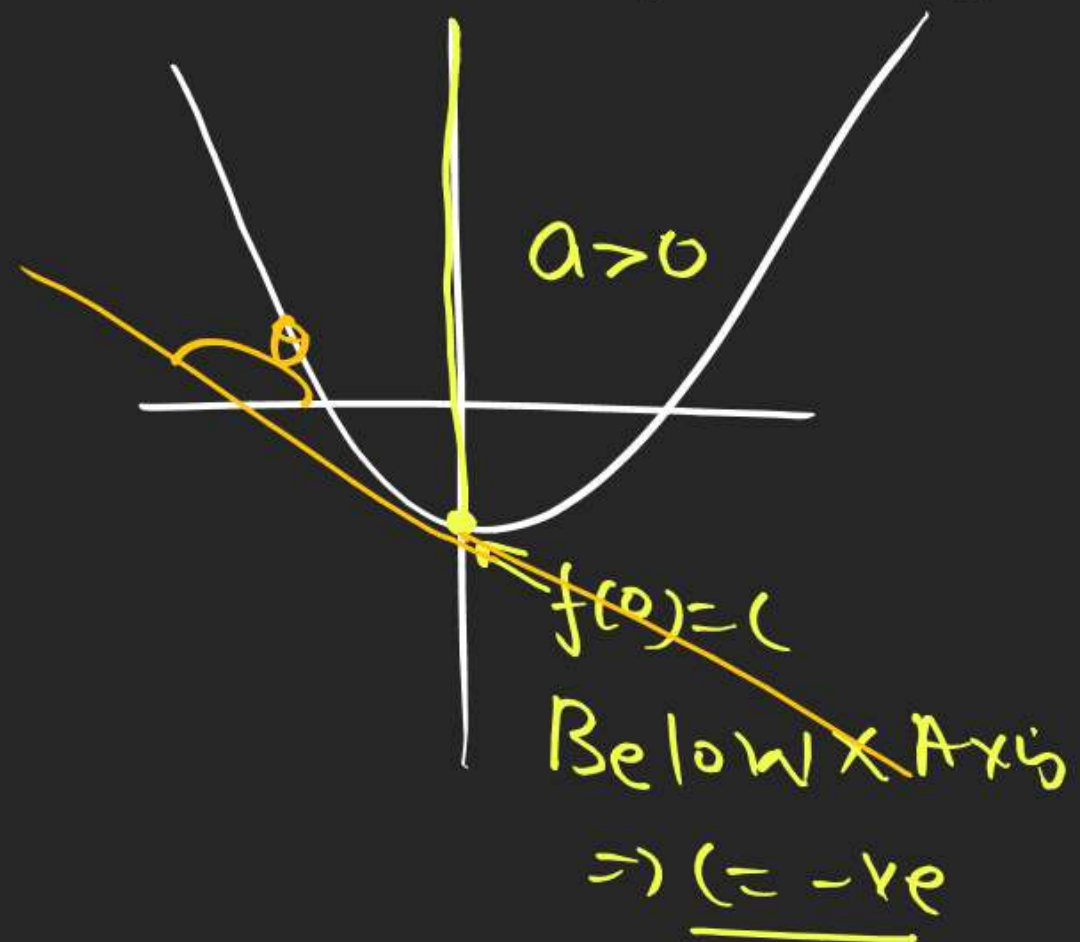
$$f(x) = ax^2 + bx + c \quad \textcircled{1} a > 0$$



$$\textcircled{2} a < 0$$

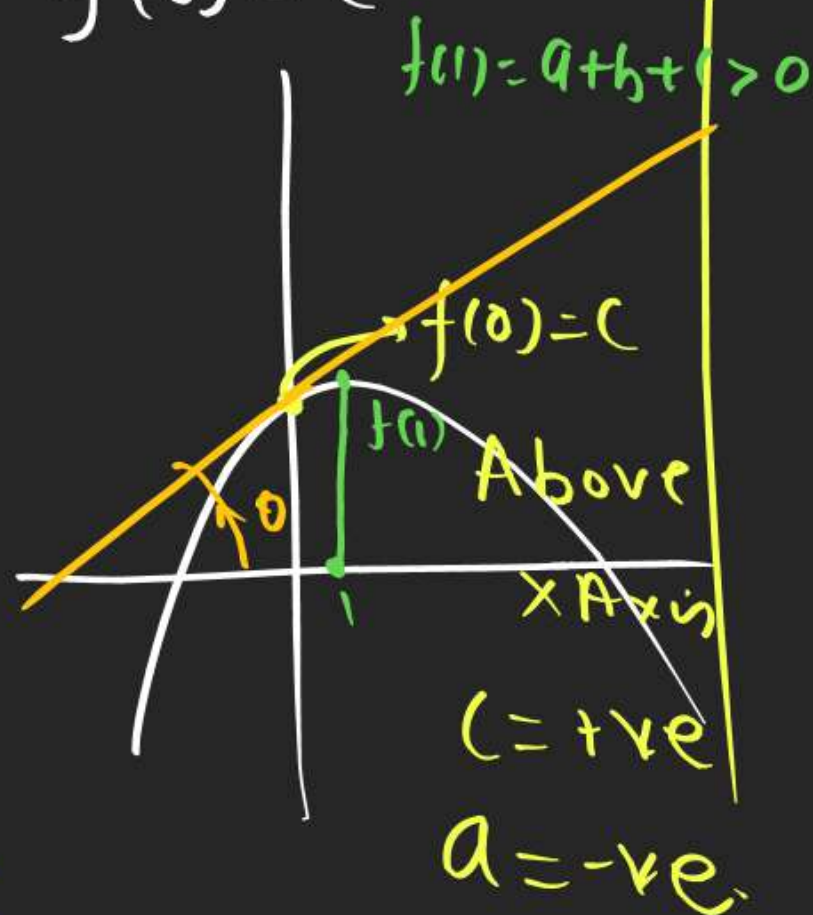


(2)* Position of c in $f(x) = ax^2 + bx + c$



$\theta = \text{obtuse angle}$
 $b = -ve$

$f(0) = c$



$\theta = \text{acute angle}$
 $b = +ve$

(3) Position of b in $f(x) = ax^2 + bx + c$

$$f'(x) = 2ax + b$$

$$f'(0) = b$$

Slope of tangent to pt where graph cuts Y Axis $= b$

(4) Sign of $a + b + c$
 (check $f(1)$)
 for $\forall x = 1 \in \mathbb{R}$ Position.

Q If $a < 0, D \leq 0$

for $f(x) = ax^2 + bx + c$

Sign of

① $4a + 2b + c = ?$

② $a^2 + c^2 - b^2 + 2ac = ?$



① Total graph
below x-axis
for all x , $f(x) < 0$

① $4a + 2b + c = f(2) = \text{below x-axis}$
 $= -ve$

② $a + b + c = f(1) = -ve$

③ $\underline{a - b + c} = f(-1) = -ve$

④ $a^2 + c^2 - b^2 + 2ac$

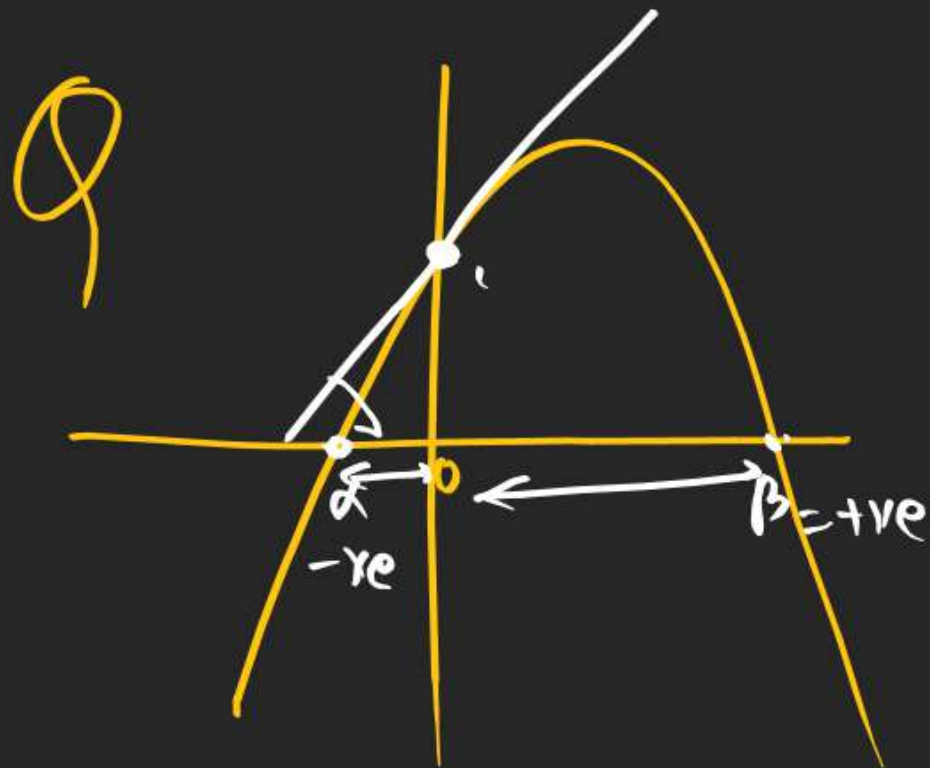
$(a^2 + c^2 + 2ac) - b^2$

$(a + c)^2 - b^2$

$= (a + c + b)(a + c - b)$

$= f(1) \times f(-1)$

$= -ve \times -ve = +ve$



Sign

(1) a = downward = -ve

(2) b = Acute = +ve

(3) $c = f(0)$ = Above x-axis = +ve

(4) D = +ve $D > 0$ Distinct Roots

(5) $POR = \alpha \cdot \beta = \ominus \oplus = \text{+ve}$

(6) $SOR = \alpha + \beta = +ve$

$$\beta > |\alpha|$$

Q For what values of a

Inequality $ax^2 + 2ax + \frac{1}{2} > 0$

$\forall x \in \mathbb{R}$.

Q fcn $\pm$ is Bole

$a = 0$

$\frac{1}{2} > 0$

$x \in \mathbb{R}$

$a = 0$

(haleg)

$a \neq 0$

Q fcn > 0

Graph above X Axis / Below X Axis

$b < 0$

$(2a)^2 - 4 \times a \times \frac{1}{2} < 0$

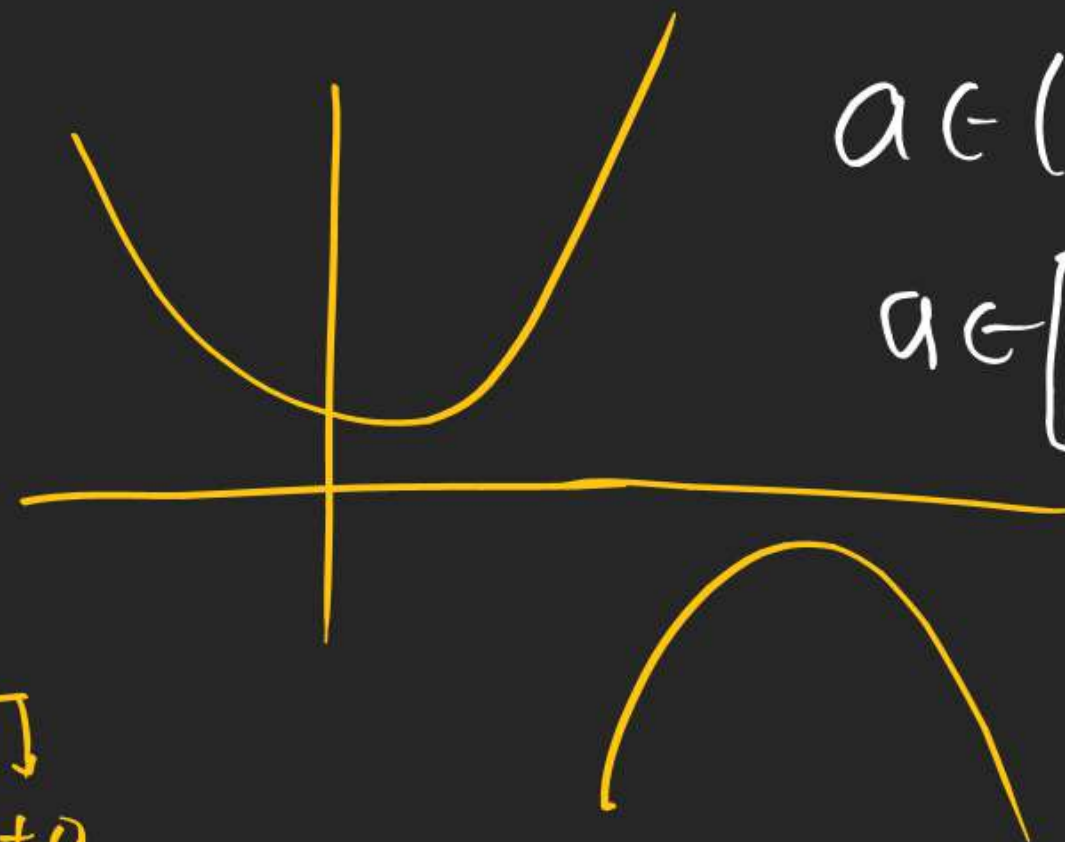
$2a^2 - a < 0$

$(a)(2a - 1) < 0$

$0 < a < \frac{1}{2}$

$a \in (0, \frac{1}{2}) \cup \{0\}$

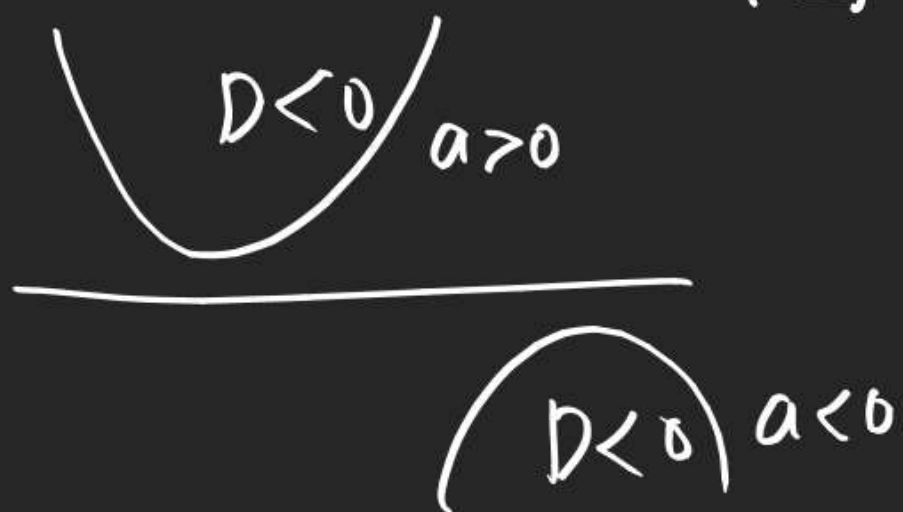
$a \in [0, \frac{1}{2})$



R_K

$$\textcircled{1} \text{ Q. fcn } ax^2+bx+c \geq 0 \rightarrow \begin{cases} a > 0 \\ D < 0 \end{cases}$$

$$\textcircled{2} \text{ Q. fcn } \underline{ax^2+bx+c} < 0 \rightarrow \begin{cases} a < 0 \\ D < 0 \end{cases}$$



Conclusion

If $D < 0$ then

Sign of Q.fcn = Sign of a if $\underline{D < 0}$

$$ax^2+bx+c \geq 0 \rightarrow a > 0$$

$$ax^2+bx+c < 0 \rightarrow a < 0$$

$$Q \text{ If } \vec{v}_1 = ax \hat{i} + \boxed{a} \hat{j} + \boxed{\frac{1}{2}} \hat{k}$$

$$\vec{v}_2 = x \hat{i} + \boxed{2x} \hat{j} + 1 \hat{k}$$

find a for which **angle**

betⁿ $\vec{v}_1$ & $\vec{v}_2$ is **acute** for each $x \in \mathbb{R}$

$$\vec{v}_1 \cdot \vec{v}_2 > 0$$

$$ax^2 + 2ax + \frac{1}{2} > 0$$

$$a \in \left[0, \frac{1}{2}\right)$$

$$Q \text{ Find } K \text{ for which } x^2 - 2(4K-1)x$$

$$+ 15K^2 - 2K - 7 > 0$$



$$\forall x \in \mathbb{R}$$

$$Q \text{ fxn} > 0$$

$$\boxed{a > 0}, \underline{b < 0}$$

$$(-2(4K-1))^2 - 4(1)(15K^2 - 2K - 7) < 0$$

$$(4K-1)^2 - (15K^2 - 2K - 7) < 0$$

$$16K^2 - 8K + 1 - 15K^2 + 2K + 7 < 0$$

$$K^2 - 6K + 8 < 0 \Rightarrow (K-2)(K-4) < 0$$

$$2 < K < 4$$

$$K \in (2, 4)$$

Q Find value of a for which

graphs of $y = 2ax + 1$ &

$y = (a-6)x^2 - 2$ does not intersect?

If graphs of 2 fcn do not intersect

$\Rightarrow$ they do not keep Real sol.

$$2ax + 1 = (a-6)x^2 - 2$$

$$(a-6)x^2 - 2ax - 3 = 0 \text{ do not have real sol.}$$

$$D < 0$$

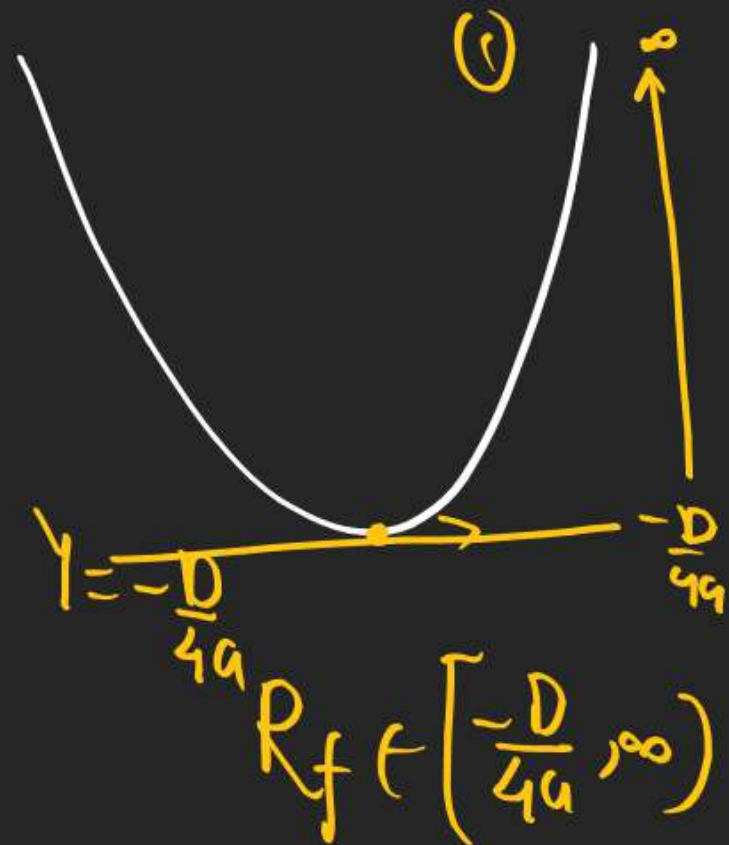
$$(-2a)^2 - 4(a-6)(-3) < 0$$

$$a^2 + 3a + 18 < 0 \Rightarrow (a+6)(a-3) < 0$$

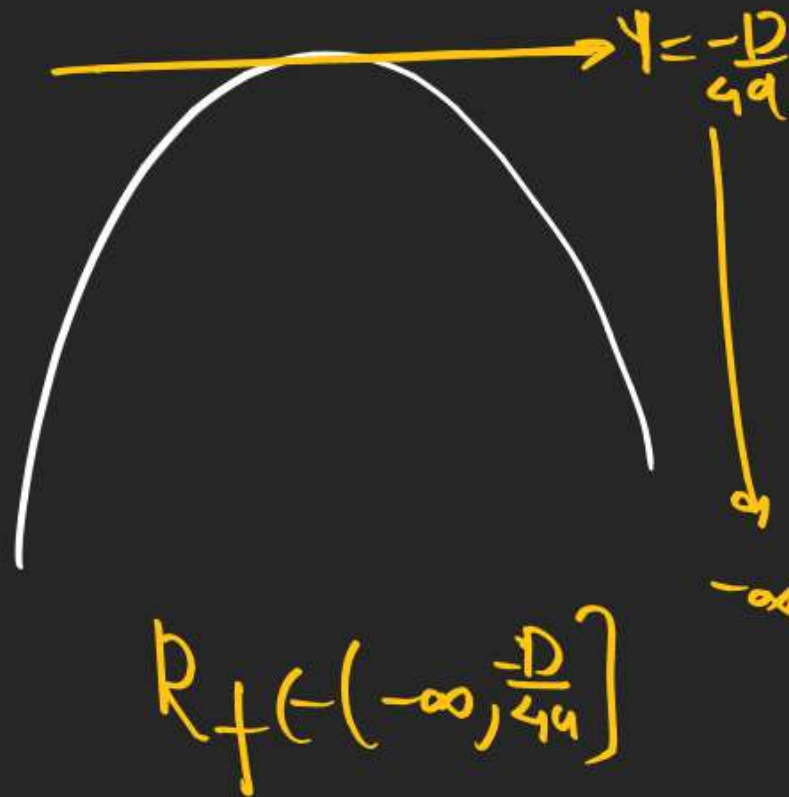
$$-6 < a < 3$$

Range of Qfn.

$$a > 0$$



$$a < 0$$



Q Range of $y = x^2 - x + 1$

$$a = 1, b = -1, c = 1$$

⊕

$$R_f \in \left[-\frac{D}{4a}, \infty\right)$$

$$\in \left[\frac{4ac - b^2}{4a}, \infty\right)$$

$$\in \left[\frac{4 \times 1 \times 1 - (-1)^2}{4 \times 1}, \infty\right)$$

$$\in \left[\frac{3}{4}, \infty\right)$$

Range of ϕ fcn when Domain is Restricted

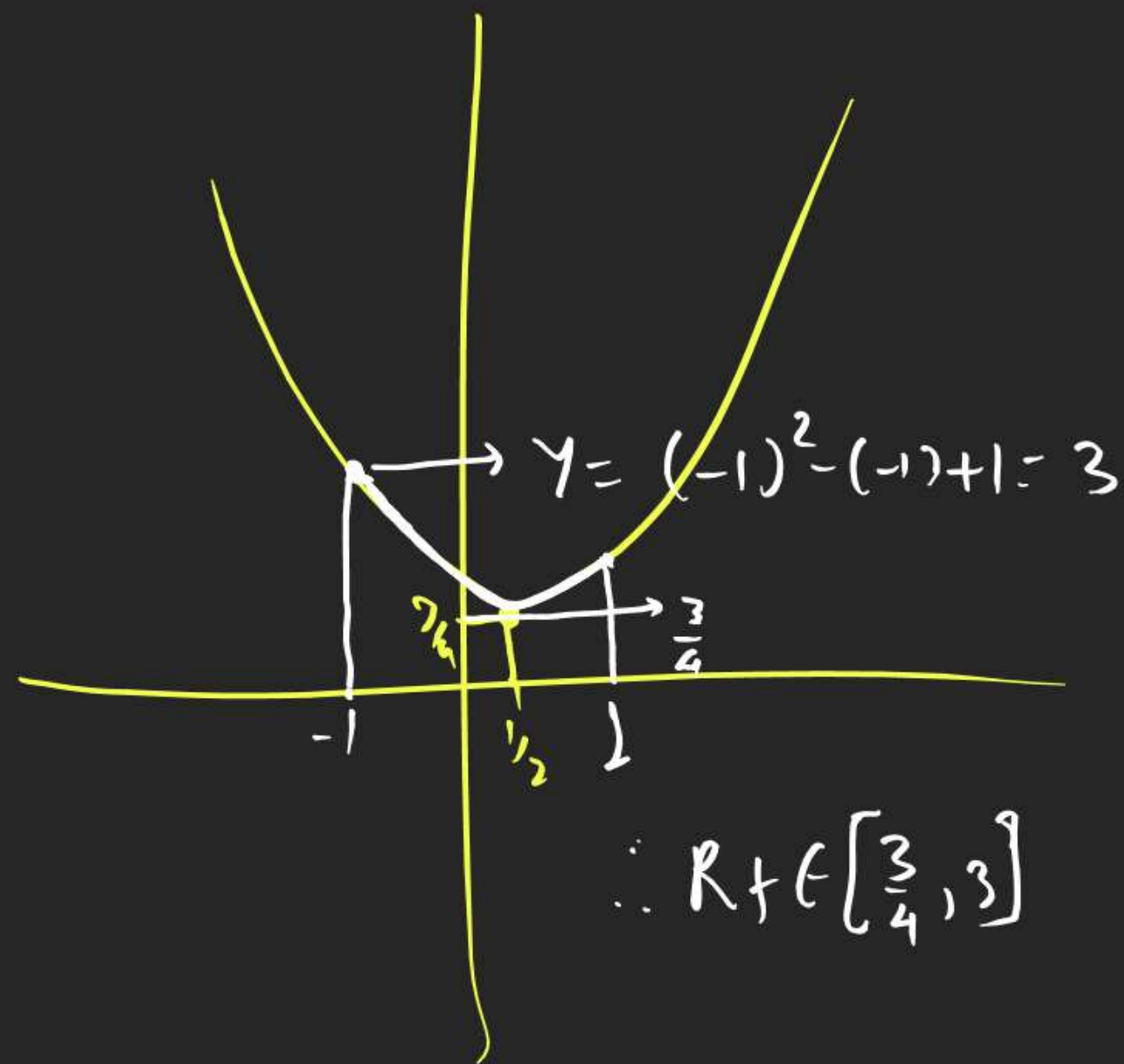
Q f(x) = $6m^2x - 6mx + 1$ find R?

$$y = t^2 - t + 1 ; t = 6mx \in [-1, 1]$$

$$\frac{dy}{dx} = 2t - 1 = 0 \Rightarrow t = \frac{1}{2}$$

$$y = \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1 = \frac{3}{4}$$

$a = 1$ & upward
Parabola

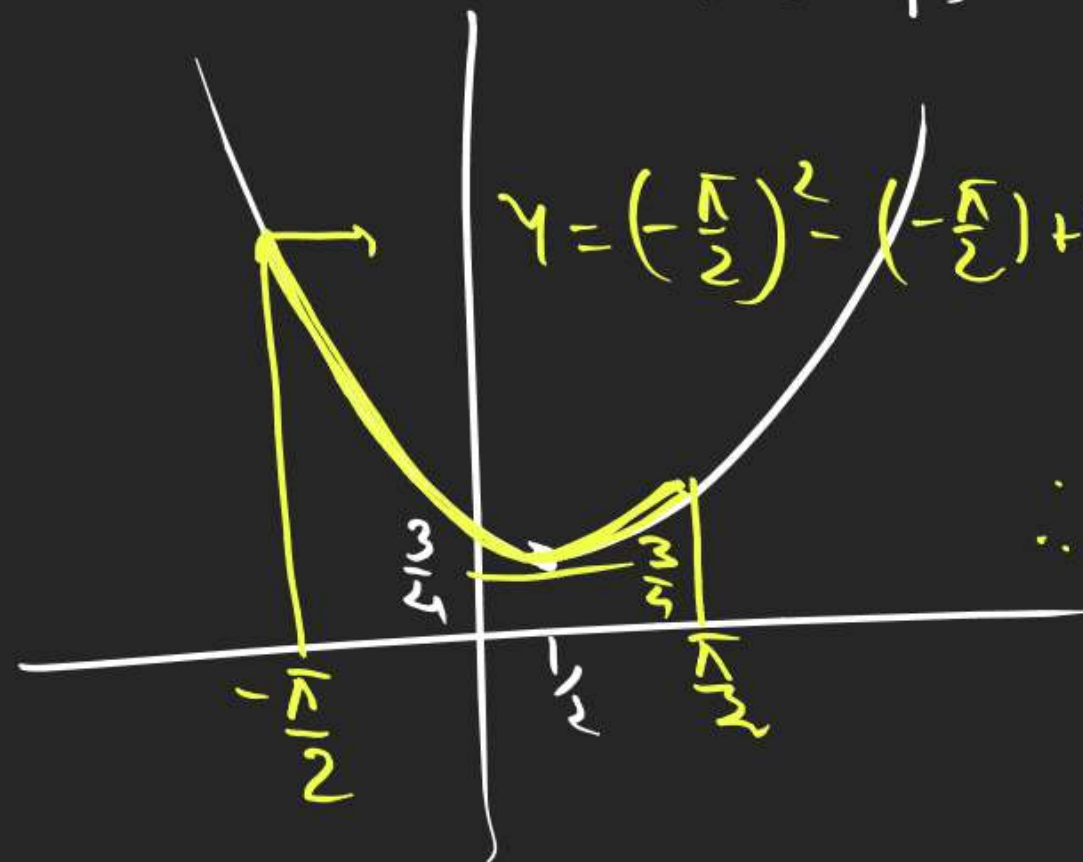


$$\therefore R = \left[\frac{3}{4}, 3\right]$$

Q Find R_f if $y = (\sin x)^2 - (\sin x) + 1$, $[-1.57, 1.57]$

$$y = t^2 - t + 1; t = \sin x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$V \equiv \left(\frac{1}{2}, \frac{3}{4}\right)$ Upward Parabola



$$y = \left(-\frac{\pi}{2}\right)^2 - \left(-\frac{\pi}{2}\right) + 1 = \frac{\pi^2}{4} + \frac{\pi}{2} + 1$$

$$\therefore R_f \in \left[\frac{3}{4}, \frac{\pi^2}{4} + \frac{\pi}{2} + 1\right]$$